CONJUGATE PROBLEMS OF NATURAL CONVECTION

L. B. Gdalevich and V. E. Fertman UDC 536.25

The increase in the number of reports on the investigation of the laws of convective motion and heat exchange under the conditions of thermal natural convection (NC) is due to the prevalence of the NC process in various engineering devices. The optimum conditions of operation of an electronic apparatus [1], of thermodiffusion fractionating columns [2], of apparatus for the sterilization of food products [3], and of cooling systems for various technological units [4-5] are determined to a large extent by NC. The problem of the boundary conditions at a solid–fluid interface is important in engineering calculations of a thermal process.

The preliminary assignment of the boundary conditions (the temperature of a surface or the heat flux through an interface) is not always satisfactory. For example, in the presence of intense heat exchange it does not reflect the important degree of connection between the heat conduction in the solid and the convection in the fluid flowing over it. With transient heat exchange the law of time variation of the surface temperature is not known in advance. With steady heat exchange the assignment of the surface temperature is justified only in the case of infinite thermal conductivity of the solid, whereas the true physical situation corresponds to finite values of the thermal conductivity and wall thickness. Therefore, in thermal calculations and the construction of engineering devices for which the thermal conditions are characterized by a significant thermal interaction between the boundaries of the structure and the fluid, it is desirable to set up the thermal problem as a conjugate problem; i.e., to seek a joint solution of the equations of fluid convection and the equation of heat conduction in the solid with equality of the previously unknown temperatures and heat fluxes at the phase interface [6].

The aim of the present survey is to discuss the methods, the specifics, and the principal results obtained in the solution of conjugate problems of NC.

1. External Problems. Of the work on heat exchange during NC at a vertical wall we will dwell on the reports [7-11]. Zinnes [12] developed a method making it possible to solve by iteration the conjugate problem of two-dimensional laminar NC at a thermally conducting plate ($0 \leqslant X \leqslant 1$, $-b/a \leqslant Y \leqslant 0$) with a heat source $G_s(X)$ distributed arbitrarily over its surface ($0 \leqslant X \leqslant 1$, $Y = 0$) [7]. The condition of equality of the heat fluxes at the solid–fluid interface,

$$\frac{k_s}{k_f} \cdot \frac{\partial \Theta_s}{\partial Y} = \frac{\partial \Theta_f}{\partial Y} + G_s(X), \quad Y = 0, \quad 0 \leqslant X \leqslant 1,$$

comprised the connection between the boundary-layer equations written in finite-difference form and the two-dimensional equation of heat conduction in the solid.

The system was solved numerically by the method of successive approximations. Numerical and physical experiments performed on glass and ceramic plates with local ribbon heat sources showed that (Fig. 1): 1) When $(k_s/k_f) \to 1$ heat removal through the plate is insignificant and NC makes the main contribution to the heat exchange, which can lead to local inversion of the heat flux; 2) when $(k_s/k_f) \geqslant 5000$ the solid–fluid thermal interaction is close in character to the heat exchange of an isothermal vertical surface, so that when $(k_s/k_f) \geqslant 5000$ the solution of the conjugate problem of NC can be replaced by calculation of the heat exchange of an isothermal surface; 3) in the region of $(k_s/k_f) < 5000$ one observes coupling between the heat exchange in the solid and the boundary layer, which makes the conjugate statement of the NC problem desirable; 4) the assumption that the coefficient of heat exchange is constant is limited and one must allow for the prehistory of the flow, which is confirmed by the region of negative values of the local Nusselt number in experiments on glass plates (Fig. 1b).

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 3, pp. 539–547, September, 1977. Original article submitted July 13, 1976.

 0022-0841/77/3303-1120$07.50

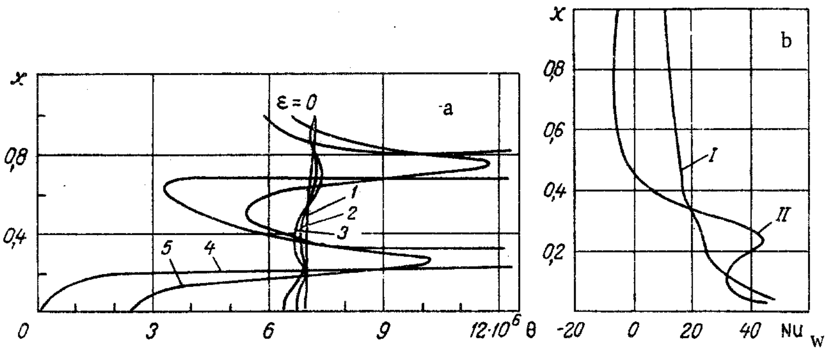

Fig. 1. a) Effect of ratio of thermal conductivities on natural convection from a vertical plate; a: 1) k_s/k_f = 5000; 2) 2500; 3) 1100; 4) 1; 5) 40; b: local Nusselt number [ceramic (I) and glass (II) plates with inclusion of a bottom heater].

Numerical methods are also effective in the solution of a class of problems on the heat conduction of a radiating plate heated at the base and the NC of a compressible fluid at its surface when the heat conduction in the plate is one-dimensional and the heat exchange in the boundary layer is two-dimensional (consistent boundary condition of the third kind) [8]. The system of equations was solved by the method of successive approximations. In each step the boundary-layer equations were replaced by a system of finite-difference equations which were solved by the trial-run method. The solution of the boundary-layer equations for some value of the wall temperature $\Theta_s(\xi)$ was taken as the initial approximation, which made it possible to calculate the generalized coefficient of heat transfer $(\partial\vartheta/\partial\eta)_0$, which was then introduced into the equation of heat conduction in the solid; and by solving it (also by the finite-difference method), the next value of $\Theta_s(\xi)$ needed for the next iteration was obtained, and so forth, until a given value of the discrepancy was reached. Experiments conducted on a steel plate (660 × 100 × 10 mm, Kh18N10T steel) with NC in air confirm the results of the numerical calculations ($Pr = 0.7$, $Ra_x \leqslant 5 \cdot 10^9$, h_{pl} = 450-723 mm, solid: Kh18N10T steel) that there is considerable lack of self-similarity of the velocity and temperature fields, a sharp decline in the generalized coefficient of heat transfer $(\partial\vartheta/\partial\eta)_0$ with height, and a considerable contribution of radiation (decisive in the upper part of the plate) to the heat exchange.

Kelleher and Yang [9], who used the best features of a method of analysis of conjugate heat exchange proposed by Perelman [13], obtained an exact analytical solution to the problem of the heat exchange of a plate submerged in an incompressible fluid ($0 \leqslant x \leqslant \infty$, $-2L \leqslant y \leqslant 0$) and heated by heat sources $Q(x, y)$ arbitrarily distributed inside it and symmetrically cooled by NC. As in [13], following a solution by a Fourier transform of the Poisson equation $\nabla^2 T_s = Q(x, y)$ the temperature distribution in the solid was expressed through the function $p(x) = [\partial T_s(x, y)/\partial y]_{y=0}$, which is not known in advance. In particular, the temperature of the interface is

$$T_w(x) = (2/\pi)\int_0^\infty p(z)\cdot \ln[1-\exp(-\pi(x+z))]\,dz - \int_0^x (z-x)\,Q(z)\,dz + \int_0^x (z-x)\,p(z)\,dz - x\int_0^\infty [Q(z)-p(z)]\,dz.$$

In [13] the first integral was omitted, since the region of the solution is large x, whereas in [9] the region adjacent to the leading edge is also taken into account. The source $G_\infty Q(x) = \sum_{k=0}^{\infty} Q_k x^{k/4}$ is a bounded analytical function as $x \to \infty$. The matching of the solution in the solid with the solution of the boundary-layer equations, obtained with the help of series of the Gertler type [14] and allowing for the nonisothermicity of the surface, determines p(x):

$$p(x) = \left(\frac{k_f}{k_s G_\infty}\right) x^{(5n-1)/4} \sum_{k=0}^{\infty} M_k x^{k/4}.$$

The effect of the ratio (k_f/k_s) and the intensity of the internal sources on the heat exchange is illustrated by a calculation for two values of $(k_f/k_s) = 0.01$ and 0.05 with $Pr = 10$, $Q|_{x>1}(x) \equiv 0$, $Q_0 = 0$, $Q_1 = Q_M = \text{const}$, $Q_2 = -Q_M$, $Q_k = 0 (k > 2)$, $Q_M = -10^4$ and -10^6. An increase in (k_f/k_s) with a constant source leads to a decrease in the temperature of the surface (Fig. 2a). The heat flux, which is not bounded at $x = 0$, decreases with an increase in x for some time (as in the case of an isothermal plate), but owing to the generation of heat in the solid it begins to grow again, having first passed through a minimum (Fig. 2b).

On the basis of an analytical solution, Bal Krishnan [10] made a graphic analysis of nonsteady heat exchange of a thin plate $(-\infty \leqslant x \leqslant \infty,\ -l \leqslant y \leqslant 0)$ in contact with an incompressible fluid $(y \geqslant 0;\ -\infty \leqslant x \leqslant \infty)$. The development of NC at the wall produced a stepwise change at $t = 0$ in the heat flux at the free surface $y = -l$ (or in the surface temperature $T_s|_{y=-l}$). The joint solution (with boundary conditions of the fourth kind) of the boundary-layer equations and the equation of heat conduction of the solid for small t was performed by a Laplace transform. The analytical expressions for the temperature and velocity fields depend on the ratio (a_s/a_f) of the thermal diffusivities of the solid and fluid. An increase in $(a_s/a_f)^{1/2}$ leads to an increase in the velocity field.

Lock and Gunn [15] analyzed the problem of the heat conduction of a thin vertical fin with a profile $\delta = \delta_R(x/\omega)^m$ with NC of a nonmetallic fluid $(Pr = \infty)$ at its surface. The NC is analyzed in a boundary-layer approximation, which is possible on the assumption of a large relative elongation ω/δ_R of the fin. Experiments show that an elongation sufficient for the neglect of the curvature and slope of the fin already occurs when $\omega = 6\delta_R$ if the ratio of thermal conductivities is $(k_s/k_f) >> 1$ [15]. In the case of a nonisothermal surface under consideration, the use of an analog of a Lefevre transformation [16] which is acceptable for $Pr = \infty$ leads to neglect of the inertial terms and replacement of the boundary-layer equations by a system of ordinary differential equations independent of the Prandtl number. The equations of the system were solved by the Meksyn method [17] with subsequent calculation on a computer. As in [8], the coefficient of convective heat exchange calculated on the basis of a self-similar solution for a power-law temperature distribution was introduced into the equation of heat conduction of the fin, while the temperature distribution of the surface of the fin was sought in the form $\Theta_\omega = \text{const}\ (x/\omega)^n$, $n > 0$, which was achieved by matching the temperature fields in the fluid and solid. The results of experiments (on water and glycerin) and theoretical calculations (the series were cut off at the seventh term), which were in good agreement, show that: 1) The effect of the Prandtl number and the shape of the fin profile on the heat transfer is slight; 2) the heat transfer is actually determined by a single dimensionless complex, the matching parameter

$$\varkappa = \left(\frac{k_s}{k_f}\right)\left(\frac{w}{\delta_R}\right)\left(\frac{\beta g \Theta_R w^3}{a_f^2\,(1+\mathrm{Pr})}\right)^{1/4},$$

which allows for the combined effect of the ratio of thermal conductivities of the media (k_f/k_s), the relative elongation of the fin (ω/δ_R), and the "motion potential" ($\Theta_R = \text{const}$ and a_f is the coefficient of thermal conductivity of the fluid)

$$\beta\Theta_R g w^3/\,[a_f^2\,(1+\mathrm{Pr})] = \begin{cases} \mathrm{Pr}\cdot\mathrm{Gr}, & \mathrm{Pr}\to\infty, \\ \mathrm{Pr}^2\cdot\mathrm{Gr}, & \mathrm{Pr}\to 0. \end{cases}$$

3) the steepness of the temperature profile in the fluid grows with an increase in the exponent n; 4) a dependence of the matching parameter $\varkappa$ on the exponent n is found.

The analysis of conjugate heat exchange with NC at the surface of a downward projecting fin submerged in an isothermal fluid [15] was extended by Lock and Ko [5] to rotating systems, where the mass force field is nonuniform, as is known. A tapering, radially oriented, thin fin of triangular shape immersed in air and rotating at a high velocity served as the model. The development of NC is connected with the transfer to the surrounding air of heat from the fin which is heated at the base. The non-self-similar system of boundary-layer equations was solved numerically through a transformation of the Blasius–Howarth type [18] $(x = X/L,\ y = YRa^{1/4}/L \Rightarrow \xi = \gamma x,\ \eta = [F_1(\xi)/(F_2(\xi)/\gamma)^{1/4}]y)$ and the introduction of the concept of "local self-similarity" [19]. The nonlinear one-dimensional equation of heat conduction of the fin was integrated by the modified Runge–Kutta method. The joining of the solutions in the boundary layer and the solid is analogous to that of [8]. The calculations showed that: 1) The temperature and velocity fields depend on the dimensionless complex

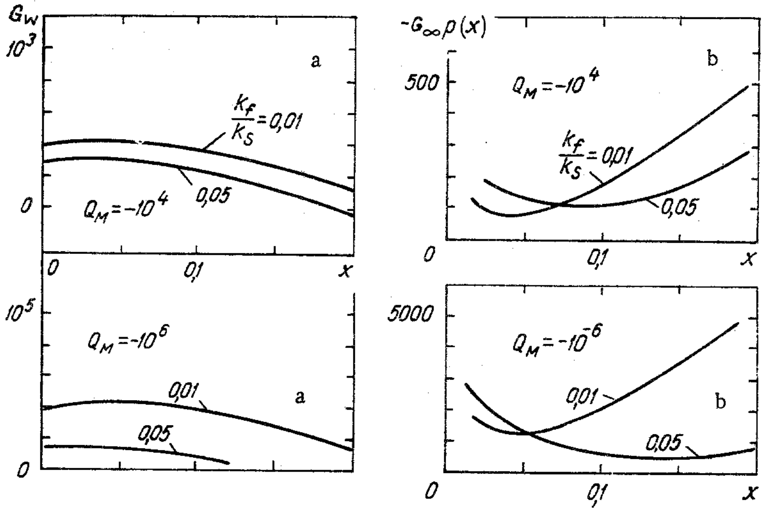

Fig. 2. a) Variation of surface temperature of solid; b) variation of heat flux through surface of solid.

$\varkappa = (\omega/\delta_R)\ (k_f/k_s)\cdot Ra^{1/4}$ [see [15] for (ω/δ_R)]; 2) the dependence $Nu/Ra^{1/4} = F(\varkappa, \gamma)$ holds for the calculation of the Nusselt number along the base of the fin ($\gamma = \omega/R$ and R is the distance from the top of the fin to the center of rotation) and the dependence of $Nu/Ra^{1/4}$ on γ has a linear character within the limits of the calculations ($\varkappa = 0.01, 0.1, 1.0, 3.0, 5.0, 10.0$; $0 \leqslant \gamma \leqslant 0.32$); 3) the efficiency of the fin declines with an increase in the matching parameter $\varkappa$, with the efficiency of a rotating fin being considerably higher than the efficiency of a fixed fin [15]; 4) the departure of the temperature field from the self-similar case is small.

The report of Lock and Ko [11] is evidently the sole report devoted to the conjugate heat exchange of fluids separated by a thermally conducting partition with NC (the analogous problem in the case of forced convection has been studied by a number of authors [20–22]). According to the method proposed in [11], the system of determining equations is reduced by a special transformation to a form convenient for the application of the method of local self-similarity [19]. The solutions depend on

$$\alpha = \frac{k_{f2}}{k_{f1}}\left(\frac{Ra_2}{Ra_1}\right)^{1/4}\left[\frac{Pr_2\,(1+Pr_1)}{Pr_1\,(1+Pr_2)}\right]^{1/4},$$

$$\varkappa = \frac{W}{L}\cdot\frac{k_{f2}}{k_s}\,Ra_2^{1/4}\left(\frac{Pr_2}{1+Pr_2}\right)^{1/4}.$$

The velocity and temperature distributions in the fluids and the temperature and heat-flux distributions in the partition are obtained numerically for the case when there is air at different temperatures on the two sides of the partition ($Pr_1 = Pr_2 = 0.72$, $\alpha = 1$).

2. Internal Problems. Vu Zuy Quang [23] studied the effect of the finite thickness and finite thermal conductivity of the walls on heat exchange with NC in the case of the steady one-dimensional convection of an electrically conducting fluid filling a vertical channel (along Oy) $2l$ wide with thermally conducting walls having a thickness h when a magnetic field B_0 = const is applied perpendicular to the walls and an alternating electric current flows along Oz (inductionless approximation, $E_z \gg E_{z\,in}$). A numerical analysis made of the solutions of the system of MHD equations of the fourth kind at the interface revealed the effect on the heat exchange of the conjugate parameter $\Psi = (k_f/k_s)\cdot(h/l)$, the parameter S characterizing the ratio of Joule heat to the heat transmitted by thermal conductivity, and the parameter n characterizing the ratio of the half-width of the channel to the thickness of the electrical skin layer. Thus, the intensification of convection and heat exchange which occurs is due to the allowance for the finite thermal conductivity and finite thickness of the walls.

1) The case of S = 0 and Ψ = 0 corresponds to NC in a vertical channel with constant and different wall temperatures. 2) The temperature and velocity increase with an increase in Ψ when $n \to \infty$ and S is fixed; i.e., the use of the conjugation conditions leads to a de-

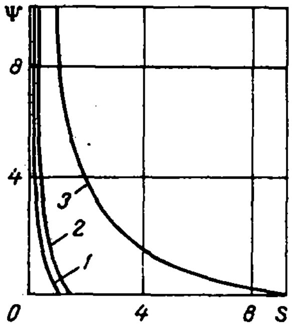

Fig. 3. Dependence of Ψ on S with $Q/(Q|_{\Psi=0}) = 0.01$ for different n. Curves 1, 2, and 3 correspond to n = 0, 1, and 5.

crease in the heat transfer from the walls. Even with small S the convection is mainly due to the release of Joule heat. 3) When $\Psi \geqslant 1$, $0.15 \leqslant S < 10$, and $n < 10$, the convection excited by the high-frequency electric current predominates over NC. 4) The dependences of Ψ on S with a ratio of heat fluxes $Q/(Q|_{\Psi=0}) = 0.01$, obtained for different n (Fig. 3), indicate a region of the parameters (to the left of the curves in the graph) where the effect of the boundary conditions of the fourth kind on the heat exchange is small.

Rotem [4] analyzed steady NC in the gap between two coaxial horizontal cylinders ($R/r < \infty$) infinitely long containing sources distributed uniformly through the volume of the inner cylinder (the core) or concentrated linearly along its axis. Natural convection develops in the gap with any heating of the surface of the core, since the vector of the gravitational force forms an angle with the temperature gradient directed radially. The temperature and velocity fields were obtained in the form of asymptotic expansions by powers of the Grashof number. The form of the expansions was established in detail and corresponds to expansions of the Stokes type [24]. Convergence of the series is guaranteed up to $Gr = 10^4$ when $Pr = 1$. The solution obtained was analyzed numerically (for different values of k_s/k_f, R/r, Gr, and Pr). The effect of conjugation appeared as follows: The isotherms which run from the liquid into the core experience a disruption in smoothness in passing through the phase interface (r = 1). From the boundary condition of equality of fluxes

$$\left(\frac{\partial \Theta_f}{\partial r}\right)_{r=1} = \left(\frac{k_s}{k_f}\right)\left(\frac{\partial \Theta_s}{\partial r}\right)_{r=1}$$

it is clear that the bend in the isotherms at r = 1 is larger, the smaller the ratio k_s/k_f of coefficients of thermal conductivity. The isotherm corresponding to $\Theta_f = 0$ no longer coincides with the contour of the core but penetrates into it. The concentricity of the core isotherms relative to the geometrical center is lost (Fig. 4).

Lau and Rotem [25] obtained the solution of the conjugate problem of NC in the gap between concentric conducting spheres ($R/r < \infty$) when the coefficient of thermal conductivity of the outer sphere is infinite while that of the inner sphere is finite. The solution was obtained in the form of binary series of powers of the Grashof and Prandtl numbers and was axisymmetric relative to the gravitational field. An analysis of the solution shows that the appearance of multivortex structures is eliminated within the region of convergence of the series.

Iqbal, Khatry, and Aggarwala [26] obtained a general solution by the variation method to the problem of the fully developed laminar mixed convection in a vertical channel of

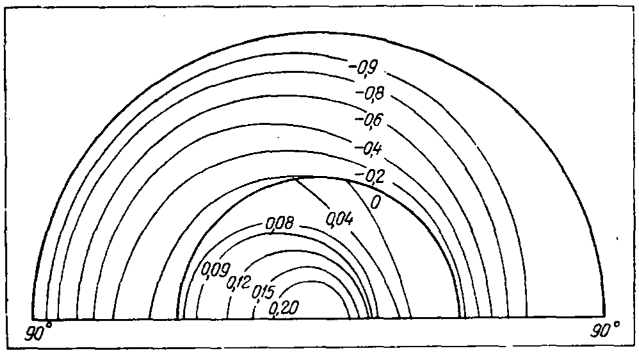

Fig. 4. Isotherms (Gr = 2800, Pr = 0.72, k_s/k_f = 10).

arbitrary shape with heat sources uniformly distributed in a wall of constant thickness δ. The conditions for joining the solutions in the channel and the wall were assigned at all the boundaries and obtained from the assumption of equality of temperatures at the inner wall and writing the equations of heat balance for an element of the wall. The problem was reduced to a search for those values of V and φ (the axial velocity and the temperature difference of the wall) which provide for the steadiness of some functional which allows for the conjugate nature of the problem. The method of approximate analytical solution of conjugate problems of this class proposed in [26] is very important, since the mathematical difficulties of obtaining an exact solution grow considerably with the condition of joining the solutions along all the boundaries.

The variation of the wall temperature and heat flux along the perimeter is determined by the conjugate parameter $K = \delta k_s/D_h k_f$ (D_h is the hydraulic diameter), which expresses the ratio of the local Nusselt number to the Biot number. An almost constant wall temperature is established as $K \to \infty$ (for channels of square cross section $\varphi \approx 0$ over the entire perimeter when $Ra = 10^3$ and $K = 20$). As $K \to 0$ the heat flux at the wall becomes constant over the perimeter. Calculations made for channels of rectangular cross section with ratios of the sides of 1, 2, and 3 show that the effect of the parameter K on the Nusselt number grows with an increase in the ratio of the sides; NC and the conjugate parameter K can considerably reduce the asymmetry in the temperature distribution along the perimeter of the wall.

3. Convective Instability. A systematic presentation of the results of the study of convective stability (including the conjugate statement) published up to 1971 is contained in [27]. Therefore, we will only discuss two reports which came out in recent years.

Vu Zuy Quang [28] studied the stability of the equilibrium of a horizontal layer of electrically conducting fluid which is in a transverse magnetic field (B_0 = const) and is confined by walls of finite thickness b_s and finite thermal conductivity k_s when the instability is due to Joule heat release. The system of equations of small monotonic perturbations of equilibrium was solved by a variant of the Bubnov–Galerkin method and the thermal boundary conditions of $dT_f/dz = (T_f - 1)/\Psi$ at $z = 0$ and $dT_f/dz = -T_f/\Psi$ at $z = 1$ were borrowed from [29]. It was established that the critical Rayleigh number $Ra_{cr} = Ra_{cr}(\alpha, Ha, \Psi, S)$ is based on $\Delta T = T_1 - T_2$ ($T_{1,2}$ are the temperatures of the outer wall surfaces), where α is the wave number, Ha is the Hartmann number, $\Psi = k_f b_s/k_s b_f$ is the conjugation parameter, and S is the internal heating. The calculations show that an increase in Ψ hardly changes Ra_{cr} in the region of $Ha = 0$, $3 \leqslant S < 10$, and $\Psi \geqslant 3$. Because of Joule heat release a region of values of the parameters of the problem appears ($Ha = 0$, $S \geqslant 10$, $\Psi > 3$) where $Ra_{cr} < 0$ when boundary conditions of the fourth kind are used. With $Ha \neq 0$ and S fixed, Ra_{cr} grows monotonically in proportion to Ψ and Ha. The strong influence of the finite thickness and finite thermal conductivity of the walls on the heat exchange appears in the region of $S \geqslant 10$ and $\Psi \geqslant 3$.

Catton [30] studied the effect of different thicknesses and thermal conductivities of the side walls of a rectangular parallelepiped on the equilibrium stability of the fluid filling it when a constant temperature gradient directed vertically downward was maintained in it. Averaging of the three-dimensional equation of heat conduction over the thickness of the wall with allowance for boundary conditions of the fourth kind leads to a new thermal boundary condition. The eigenvalue problem obtained on the basis of the theory of stability (Ra is the eigenvalue) was solved by Galerkin's method. The calculations made for $0 < k_s \leqslant \infty$ and ratios of the width of the cavity to its depth of from 1/3 to 8 reveal a dependence of Ra_{cr} on the parameters $C_x = k_f L/(k_s\delta)_x$ and $C_y = k_f L/(k_s\delta)_y$ (L is the height of the parallelepiped and δ is the wall thickness) and the shape of the cross section. The parameters C_x and C_y can have a strong effect on Ra_{cr}: A 20-fold increase in Ra_{cr} was achieved through variations in C_x and C_y for a certain shape of the cavity cross section.

The use of numerical methods of solving the initial systems of differential equations in partial derivatives with the appropriate boundary conditions is evidently the most promising in studies of conjugate NC, which is indicated by the experience already accumulated in the solution of NC problems in a conjugate statement, which shows that the expenditures of computer time on the solution of a conjugate problem in a number of cases [5, 7, 31] do not much exceed the time required for the solution of the corresponding nonconjugate boundary-layer problem.

LITERATURE CITED

1. G. N. Dul'nev and N. N. Tarnovskii, Thermal Regimes of Electronic Apparatus [in Russian], Énergiya, Leningrad (1971).
2. G. D. Rabinovich, M. A. Bukhtilova, and Zh. V. Lepekhina, Inzh.-Fiz. Zh., 28, No. 6, 1099-1128 (1975).
3. J. Hiddink, Agr. Res. Reps., No. 839, 128 (1975).
4. Z. Rotem, Int. J. Heat Mass Transfer, 15, No. 9, 1679-1693 (1972).
5. G. S. H. Lock and R. S. Ko, J. Heat Transfer, Trans. ASME, C94, No. 4, 419-424 (1972).
6. A. V. Lykov and T. L. Perel'man, Heat and Mass Exchange of Bodies with the Surrounding Gas Medium [in Russian], Nauka i Tekhnika, Minsk (1965).
7. A. E. Zinnes, J. Heat Transfer, Trans. ASME, C92, No. 3, 528-535 (1970).
8. A. M. Andreev, V. N. Piskunov, Yu. A. Sokovishin, and V. S. Yuferev, in: Thermal Gas Lenses and Thermohydrodynamic Light Guides [in Russian], Minsk (1974), p. 168.
9. M. D. Kelleher and K.-T. Yang, Appl. Sci. Res., 17, Nos. 4-5, 249-269 (1967).
10. Bal Krishnan, Indian J. Pure Appl. Math., 4, No. 4, 449-460 (1973).
11. G. S. H. Lock and R. S. Ko, Int. J. Heat Mass Transfer, 16, No. 11, 2087-2096 (1973).
12. A. E. Zinnes, Doctoral Dissertation, Lehigh University, Bethlehem, Pa. (1969).
13. T. L. Perelman, Int. J. Heat Mass Transfer, 3, No. 4, 293-303 (1961).
14. M. D. Kelleher and K.-T. Yang, Quart. J. Mech. Appl. Math., 25, No. 4, 447-457 (1972).
15. G. S. H. Lock and J. C. Gunn, J. Heat Transfer, Trans. ASME, C90, No. 1, 63-70 (1968).
16. E. J. Lefevre, Heat Division Paper 113, Mech. Eng. Res. Lab., Dept. Sci. Eng. Res., Glasgow (1956).
17. D. Meksyn, New Methods in Laminar Boundary Layer Theory, Pergamon Press, London (1961).
18. L. Howarth, Rep. Memor. Res. Coun. London, No. 1632 (1934-1935).
19. A. A. Hayday, D. A. Bowlus, and R. A. McGraw, J. Heat Transfer, Trans. ASME, C89, No. 3 (1967) (ASME Paper 66 WA/HT-17).
20. R. Viskanta and M. Abrams, Int. J. Heat Mass Transfer, 14, No. 9, 1311-1321 (1971).
21. B. M. Khusid, Inzh.-Fiz. Zh., 25, No. 1, 135-144 (1973).
22. A. Sh. Dorfman, Teplofiz. Vys. Temp., 12, No. 1, 116-120 (1975).
23. Vu Zuy Quang, Inzh.-Fiz. Zh., 26, No. 4, 747-753 (1974).
24. I. Proudman and J. R. A. Pearson, J. Fluid Mech., 2, 237-262 (1957).
25. H. M. Lau and Z. Rotem, in: Proceedings of the Fourth Canadian Congress on Applied Mechanics (CANCAM 73), Montreal (1973), pp. 785-786.
26. M. Iqbal, A. K. Khatry, and B. D. Aggarwala, J. Heat Transfer, Trans. ASME, C94, No. 1, 32-36 (1972).
27. G. Z. Gershuni and E. M. Zhukhovitskii, Convective Stability of an Incompressible Fluid [in Russian], Nauka, Moscow (1972).
28. Vu Zui Kuang, Magn. Gidrodin., No. 1, 83-88 (1974).
29. C. P. Yu and H. K. Yang, Appl. Sci. Res., 20, No. 1, 16-24 (1969).
30. I. Catton, J. Heat Transfer, Trans. ASME, C94, No. 4, 543-545 (1973).
31. V. S. Kuptsova, in: Problems of Heat Transfer [in Russian], Moscow (1976), pp. 116-124.